Causal Models

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Abstract

Notes for G8412. Some background on DAGs and questions on an argument.

1 A note on DAGs

DAGs—directed acyclic graphs—are diagrams used to represent causal structures. They can be very helpful both for clarifying an argument and for showing why some identification strategy makes sense.

The graphs are composed of a set of *nodes*—which correspond to variables (independent variables, dependent variables, and controls) with arrows between them. An arrow between A and B is used to show a possible causal relation between A and B (A exercising a causal effect on B in some conditions). The "directed" part means that if A causes B then B does not cause A. The acyclic part means that there is not set of arrows that leads from any node back to itself.

Note that arrows are not *signed*, so you cannot read off from the graph whether there is a positive or a negative causal effect. Similarly you cannot read off whether the effect is linear or interactive. Also note that arrows do not point into arrows. People want to do this sometimes to suggest that some variable modifies the effect of a second variable on a third variable but if you think about it (do!) you will see that that implies that the second variable modifies the effect of the first variable on the third variable. Better just to have both the first and the second variable pointing into the third variable to show that the third variable depends on both of these.

The ordering of the variables can be used to define family relations between variables. Arrows point into variables from the variable's "parents" and arrows point to a variable's "children." A variable's parents plus their parents (etc) are the variable's ancestors; children's children etc are "descendants."

A key assumption in causal graphs is that, given its parents, a node is independent of its non descendants. This is called the Markov property.

Figure 1 gives an example of a simple DAG.

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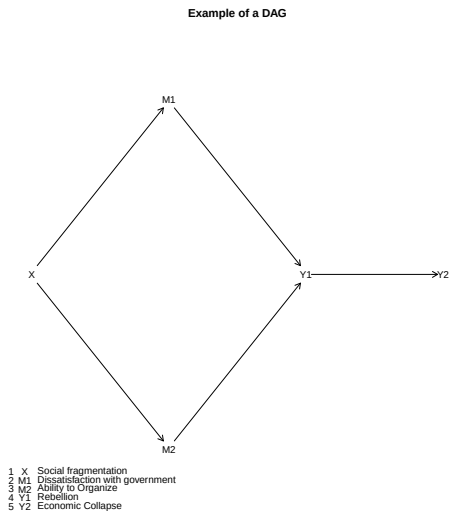

Figure 1: A Directed Acyclic Graph

1.1 Conditional independencies

A couple of things that you can read off straight from this graph:

- Y1 is not independent of M1 or M2 or X
- Y1 is not independent of X given M1
- Y1 is independent of X given M1 and M2
- M1 is independent of M2 given X
- M1 is not independent of M2 given X and Y1
- Y2 is independent of M1 given Y1

1.2 Data Generation

Lets put some flesh on this. Say data is generated like this:

- there is social fragmentation with probability .5.
- *if* there is fragmentation then the probability that a group could organize is 1/3; otherwise it is 2/3
- (independently) *if* there is fragmentation then the probability that there will be dissatisfaction is 2/3; otherwise it is 1/3
- there is a rebellion if and only if there is dissatisfaction and the ability to organize

OK. Then the distribution of outcomes might look like this. The probability of a rebellion here is $\frac{1}{2}\frac{1}{3}\frac{2}{3} + \frac{1}{2}\frac{2}{3}\frac{1}{3} = \frac{2}{9}$. The distribution of outcomes looks like this:

Prob	Social Fragmentation	Ability to organize	Dissatisfaction	Rebellion
2 in 18	0	0	0	0
1 in 18	0	0	1	0
2 in 18	1	0	0	0
4 in 18	1	0	1	0
4 in 18	0	1	0	0
2 in 18	0	1	1	1
1 in 18	1	1	0	0
2 in 18	1	1	1	1

1.3 Take Home Puzzles:

Work through these yourself (solutions below also).

Can you check if the Markov property holds?

Can you calculate:

1. "The probability of a rebellion given the ability to organize is strong."
2. "The probability of a rebellion given the ability to organize is weak."
3. "The probability of a rebellion if you *intervene* to make sure of a strong the ability to organize."
4. "The probability of a rebellion if you *intervene* to make sure of a weak ability to organize."
5. "The effect of the ability to organize on rebellion"

1.4 Solutions

For "the probability of a rebellion given the ability to organize is strong," condition on strong ability and you get 4/9.(Note that 4/9 is just the probability of dissatisfaction given ability which is less than .5 because of the negative correlation between these.)

Prob	Social Fragmentation	Ability to organize	Dissatisfaction	Rebellion
4 in 18	0	1	0	0
2 in 18	0	1	1	1
1 in 18	1	1	0	0
2 in 18	1	1	1	1

The second, "The probability of a rebellion given the ability to organize is weak" is found by conditioning on those cases in which the ability to organize is weak, as below:

Prob	Social Fragmentation	Ability to organize	Dissatisfaction	Rebellion
2 in 18	0	0	0	0
1 in 18	0	0	1	0
2 in 18	1	0	0	0
4 in 18	1	0	1	0

The answer is 0. So the difference in the probabilities is 4/9.

Imagine though now that you were intervening. In that case you would create a new distribution like this (in the strong case):

Prob	Social Fragmentation	Ability to organize	Dissatisfaction	Rebellion
2 in 18	0	1	0	0
1 in 18	0	1	1	1
2 in 18	1	1	0	0
4 in 18	1	1	1	1
4 in 18	0	1	0	0
2 in 18	0	1	1	1
1 in 18	1	1	0	0
2 in 18	1	1	1	1

In this case you expect to see rebellion with probability 9/18, or .5:

To get this table I am changing the value for "ability" and changing the value of "rebellion" accordingly, given the new value for ability and the existing value for dissatisfaction. This is using a "do" operator in the language of causal inference (see Pearl, *Causality*).

Similarly you can figure out the weak case by changing the value for "ability" to 0 and changing the value of "rebellion" to 0 everywhere accordingly.

Together these imply the causal effect is 1/2, which is not the same as the *observable* effect of 4/9.

We are led astray by the observational data because those cases in which we see an ability to organize are probably cases in which there is not much dissatisfaction and so there is not that much rebellion. Whereas when we intervene we are intervening regardless of whether there is dissatisfaction.

2 An argument:

Here is a complete, albeit barebones (and possibly incorrect), argument:

1. Good institutions (I) cause economic growth (G), except in countries that have large stocks of natural resources (N).
2. The reason is that institutions encourage people to invest (V) which spurs growth (this effect does not kick in in natural resource rich countries however since people live off rent rather than off production).
3. Growth also makes it easier to maintain good institutions, which creates a virtuous cycle.
4. Being an ally (A) of the United States also helps economic growth, but it sometimes corrupts domestic institutions.
5. Historically, places with climates (C) suitable for colonizers to settle in ended up with better institutions. Except for their effect on institutions, these climatic conditions are irrelevant for understanding contemporary economic growth. We can use C to establish the claim that I causes G.

2.1 Some counterarguments:

1. Places with climates suitable for colonizers benefited from better access to international markets which led to growth.
2. Good soil is also important for growth!
3. Good institutions also make sure that investments yield greater returns and that's what causes growth

DAG it!

2.2 Questions

Make sure you can answer all these questions:

1. What are the dependent variables?
2. What are the independent variables?
3. What are the mediating variables?
4. What are the conditioning variables?
5. What are the confounding variables?
6. What are the instrumental variables?
7. Say I and G are positively correlated. Does this mean that I causes G?
8. Say I and G are negatively correlated. Does this mean that I does not cause G?

9. How might you estimate the effect of I on G?
10. How does C help establish the link between I and G?
11. Where is the theory? Is it in equivalent to the graph or does it generate the graph?
12. How might you check if the proposed theory is correct?
13. Which of the counterarguments are strong and why?